

# Natural Two-Higgs-Doublet Model

Bohdan Grzadkowski<sup>\*1</sup> and Per Osland<sup>2</sup>

<sup>1</sup> Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Hoża 69, PL-00-681 Warsaw, Poland

<sup>2</sup> Department of Physics and Technology, University of Bergen, Postboks 7803, N-5020 Bergen, Norway

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We show that the Two-Higgs-Doublet Model (2HDM) constrained by the two-loop-order requirement of cancellation of quadratic divergences is consistent with the existing experimental constraints. The model allows to ameliorate the little hierarchy problem by suppressing the quadratic corrections to scalar masses and lifting the mass of the lightest Higgs boson. A strong source of CP violation emerges from the scalar potential. The cutoff originating from the naturalness arguments is shifted from  $\sim 0.6$  TeV in the Standard Model to  $\gtrsim 6$  TeV in the 2HDM, depending on the mass of the lightest scalar.

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## 1 Introduction

We are going to discuss an extension of the Standard Model (SM) that is free of quadratic divergences up to the leading order at the two-loop level of the perturbation expansion. The quadratic divergences were first discussed within the SM by Veltman [1], who, adopting dimensional reduction [2], found the following quadratically divergent one-loop contribution to the Higgs boson ( $h$ ) mass  $\delta^{(\text{SM})}m_h^2 = \Lambda^2/(\pi^2 v^2)(\frac{3}{2}m_t^2 - (6m_W^2 + 3m_Z^2)/8 - 3m_h^2/8)$  with  $\Lambda$  being a UV cutoff and  $v \simeq 246$  GeV denoting the vacuum expectation value of the scalar doublet. The issue of quadratic divergences was then investigated adopting other regularization schemes (e.g. point splitting [3]) and also in [4] without reference to any regularization scheme.

Precision measurements within the SM imply a small Higgs-boson mass, therefore the correction  $\delta^{(\text{SM})}m_h^2$  exceeds the mass itself even for small values of  $\Lambda$ , e.g. for  $m_h = 130$  GeV one obtains  $\delta^{(\text{SM})}m_h^2 \simeq m_h^2$  already for  $\Lambda \simeq 600$  GeV. However, if one assumes that the scale of new physics is widely separated from the electro-weak scale, then constraints that arise from analysis of operators of dimension 6 require  $\Lambda \gtrsim$  a few TeV. The conclusion that follows from this observation is that regardless of what physics lies beyond the SM, some amount of fine tuning is necessary; either one tunes to lift the cutoff above  $\Lambda \simeq 600$  GeV, or one tunes when precision observables measured at LEP are fitted. Tuning both in corrections to the Higgs mass and in LEP physics is also an acceptable alternative which we are going to explore below. In other words, we will look for new physics in the TeV range which will allow to lift the cutoff implied by quadratic corrections to  $m_h^2$  to the multi-TeV range *and* which will be consistent with all the experimental constraints—both require some amount of tuning. It should be realized that within the SM the requirement  $\delta^{(\text{SM})}m_h^2 = 0$  implies an unrealistically large Higgs boson mass  $m_h \simeq 310$  GeV.

Here we will argue that within the Two-Higgs-Doublet Model (2HDM) one can soften the little hierarchy problem both by suppressing quadratic corrections to scalar masses *and* it allows to lift the central value for the lightest Higgs mass up to a value which is well above the LEP limit.

\* Corresponding author: e-mail: bohdan.grzadkowski@fuw.edu.pl, Phone: +48 225 532 259, Fax: +48 226 219 475

## 2 The Two-Higgs-Doublet Model

In order to accommodate CP violation we consider here a 2HDM with softly broken  $\mathbb{Z}_2$  symmetry which acts as  $\Phi_1 \rightarrow -\Phi_1$  and  $u_R \rightarrow -u_R$  (all other fields are neutral). The scalar potential then reads

$$V(\phi_1, \phi_2) = -\frac{1}{2} \left\{ m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \left[ m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.} \right] \right\} + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2} \left[ \lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{H.c.} \right] \quad (1)$$

The minimum of the potential is achieved at  $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$  and  $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$ . We assume that  $\phi_1$  and  $\phi_2$  couple to down- and up-type quarks, respectively (the so-called 2HDM II).

### 2.1 Quadratic divergences

At the one-loop level the cancellation of quadratic divergences for the 2-point scalar Green's functions at zero external momenta ( $\Gamma_i$ ,  $i = 1, 2$ ) in the 2HDM type II model implies [5]

$$\Gamma_1 \equiv \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{v^2}{2} \left( \frac{3}{2} \lambda_1 + \lambda_3 + \frac{1}{2} \lambda_4 \right) - 3 \frac{m_b^2}{c_\beta^2} = 0, \quad (2)$$

$$\Gamma_2 \equiv \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{v^2}{2} \left( \frac{3}{2} \lambda_2 + \lambda_3 + \frac{1}{2} \lambda_4 \right) - 3 \frac{m_t^2}{s_\beta^2} = 0, \quad (3)$$

where  $v^2 \equiv v_1^2 + v_2^2$ ,  $\tan \beta \equiv v_2/v_1$  and we use the notation:  $s_\beta \equiv \sin \beta$  and  $c_\beta \equiv \cos \beta$ . In the type II model the mixed,  $\phi_1 - \phi_2$ , Green's function is not quadratically divergent. Some phenomenological consequences of the cancellation were discussed already in [6].

As shown in [7], the quartic couplings  $\lambda_i$  can be expressed in terms of the mass parameters and elements of the rotation matrix needed for diagonalization of the scalar masses. So, for a given choice of  $\alpha_i$ 's, the squared neutral-Higgs masses  $M_1^2$ ,  $M_2^2$  and  $M_3^2$  can be determined from the cancellation conditions (2)–(3) in terms of  $\tan \beta$ ,  $\mu^2$  and  $M_{H^\pm}^2$ . It is worth noticing that scalar masses resulting from a scan over  $\alpha_i$ ,  $M_{H^\pm}$  and  $\tan \beta$  exhibit a striking mass degeneracy in the case of large  $\tan \beta$ :  $M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$ .

At the two-loop level the leading contributions to quadratic divergences are of the form of  $\Lambda^2 \ln \Lambda$ . They could be determined adopting a strategy noticed by Einhorn and Jones [4], so that the cancellation conditions for quadratic divergences up to the leading two-loop order read:

$$\Gamma_1 + \delta\Gamma_1 = 0 \quad \text{and} \quad \Gamma_2 + \delta\Gamma_2 = 0 \quad (4)$$

with

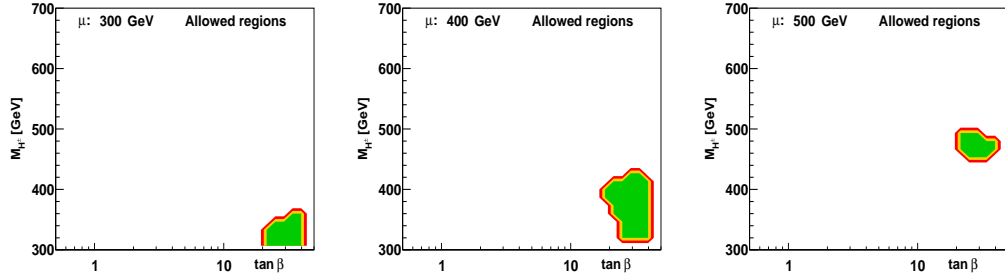
$$\delta\Gamma_1 = \frac{v^2}{8} [9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_1} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4}] \ln \left( \frac{\Lambda}{\bar{\mu}} \right) \quad (5)$$

$$\delta\Gamma_2 = \frac{v^2}{8} [9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_2} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4} - 24g_t\beta_{g_t}] \ln \left( \frac{\Lambda}{\bar{\mu}} \right) \quad (6)$$

where  $\beta$ 's are the appropriate beta functions while  $\bar{\mu}$  is the renormalization scale. We will be solving the conditions (4) for the scalar masses  $M_i^2$  for a given set of  $\alpha_i$ 's,  $\tan \beta$ ,  $\mu^2$  and  $M_{H^\pm}^2$ . For the renormalization scale we will adopt  $v$ , so  $\bar{\mu} = v$ . Those masses together with the corresponding coupling constants, will be used to find predictions of the model for various observables.

### 2.2 Allowed regions

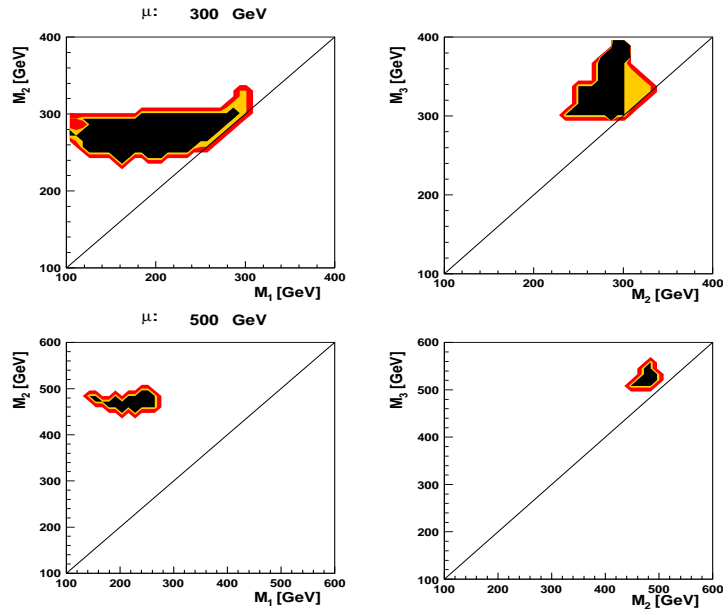
In order to find phenomenologically acceptable regions in the parameter space we impose the following experimental constraints: the oblique parameters  $T$  and  $S$ ,  $B_0 - \bar{B}_0$  mixing,  $B \rightarrow X_s \gamma$ ,  $B \rightarrow \tau \bar{\nu}_\tau X$ ,  $B \rightarrow D \tau \bar{\nu}_\tau$ , LEP2 Higgs-boson non-discovery,  $R_b$ , the muon anomalous magnetic moment and the electron



**Fig. 1** Two-loop allowed regions in the  $\tan \beta$ – $M_{H^\pm}$  plane, for  $\Lambda = 4.5$  TeV, for  $\mu = 300, 400, 500$  GeV (as indicated). Red: positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

electric dipole moment (for details concerning the experimental constraints, see refs. [8, 7, 9]). Subject to all these constraints, we find allowed solutions of (4). For instance, imposing the positivity constraints we find allowed regions in the  $\tan \beta$ – $M_{H^\pm}$  plane as illustrated by the red domains in the  $\tan \beta$ – $M_{H^\pm}$  plane, see Fig. 1 for fixed values of  $\mu$ . The allowed regions were obtained scanning over the mixing angles  $\alpha_i$  and solving the two-loop cancellation conditions (4). Imposing also unitarity in the Higgs-Higgs-scattering sector [10, 11, 12], the allowed regions are only slightly reduced (yellow regions). Requiring that also experimental constraints are satisfied the green regions are obtained.

For parameters that are consistent with unitarity, positivity, experimental constraints and the two-loop cancellation conditions (4), we show in Fig. 2 scalar masses resulting from a scan over  $\alpha_i$ ,  $M_{H^\pm}$  and  $\tan \beta$ .



**Fig. 2** Two-loop distributions of allowed masses  $M_2$  vs  $M_1$  (left panels) and  $M_3$  vs  $M_2$  (right) for  $\Lambda = 4.5$  TeV, resulting from a scan over the full range of  $\alpha_i$ ,  $\tan \beta \in (0.5, 50)$  and  $M_{H^\pm} \in (300, 700)$  GeV, for  $\mu = 300, 500$  GeV. Red: Positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

### 2.3 CP violation

CP violation could be parametrized adopting the  $U(2)$ -invariants introduced by Lavoura and Silva [13], [14]. It is convenient to use the invariants  $J_1$ ,  $J_2$  and  $J_3$  defined in [15]. The Higgs sector is CP-conserving if and only if  $\text{Im } J_i = 0$  for all  $i$  [15]. The invariants calculated in the basis adopted here could be found in [8]. As we have noticed earlier large  $\tan \beta$  implies approximate degeneracy of scalar masses. That could jeopardize the CP violation in the potential since it is well known that the exact degeneracy  $M_1 = M_2 = M_3$  results in vanishing invariants  $\text{Im } J_i$  and no CP violation (exact degeneracy implies  $\text{Im } \lambda_5 = 0$ ). Adopting the one-loop conditions (2)–(3) one easily finds that  $\lambda_1 - \lambda_2 = 4(m_b^2/c_\beta^2 - m_t^2/s_\beta^2)/v^2$ , which implies

$$\text{Im } J_1 = 4 \text{Im } \lambda_5 (c_\beta^2 m_t^2 - s_\beta^2 m_b^2)/v^2 = -4 \text{Im } \lambda_5 (m_b/v)^2 + \mathcal{O}(\text{Im } \lambda_5 / \tan^2 \beta) \quad (7)$$

If  $\tan \beta$  is large then  $\text{Im } J_1$  is reduced not only by  $\text{Im } \lambda_5 \simeq 0$  (as caused by  $M_1 \simeq M_2 \simeq M_3$ ) but also by the factor  $(m_b^2/v^2)$ , as implied by the cancellation conditions (2)–(3). The same suppression factor appears for  $\text{Im } J_3$ . The case of  $\text{Im } J_2$  is more involved, however when  $m_b^2/v^2$  is neglected all the invariants have the same behavior for large  $\tan \beta$ :

$$\text{Im } J_i \sim \text{Im } \lambda_5 / \tan^2 \beta \quad (8)$$

Those conclusions apply also at the two-loop level. It turns out that at high values of  $\tan \beta$  these invariants are of the order of  $10^{-3}$ , in qualitative agreement with the discussion above. It is worth emphasizing that the corresponding invariant in the SM;  $\text{Im } Q = \text{Im } (V_{ud} V_{cb} V_{ub}^* V_{cd}^*)$  is of the order of  $\sim 2 \times 10^{-5} \sin \delta_{KM}$  ( $V_{ij}$  and  $\delta_{KM}$  are elements of the CKM matrix and CP-violating phase, respectively). Therefore the model considered here offers much stronger CP violation than in the SM.

## 3 Summary

It has been shown that within the Two-Higgs-Doublet Model type II there exist regions of the parameter space where the quadratic divergences in scalar boson masses are suppressed. The little hierarchy problem is therefore ameliorated. The UV cutoff could be shifted up to  $\sim 6$  TeV. CP-violation emerging from the scalar potential turns out to be much stronger than within the SM.

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